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BARTŁOMIEJ ANDRZEJ MAJEWSKI
MATHEMATICS
Supervisor: Mrs Joanna Olesińska

Title

Bankers and bandits, so about counting discrete structures

Abstract

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The purpose of the essay is to examine 2 different combinatorial methods, the paths and the binary dominating sequences, by using them to solve a mathematical problem. The exercise used to conduct the investigation is:

“A pilgrim, who decides to make the voyage around the world, comes across bankers and bandits. Each met banker gives a gold bar to a pilgrim; while each bandit he meets, claims one gold bar for letting the pilgrim pass. Is it possible that for a given arrangement of $n + 1$ bankers and n , $n \in \mathbb{N}$ bandits, the pilgrim can choose a starting point, so that he will be capable of paying to each bandit? Is it possible he has at least one bar of gold through the whole journey?”

The essay provides information and explanation for two methods, as well as proofs of the formulas and lemmas used to support the argument and solve the exercise. The research question problem is solved using each of the methods, firstly with a slight modification, and then in the original form, in order to deepen the understanding and broaden the scope of the analysis.

The paper reveals that the number of ways a pilgrim may choose, in order to be capable of paying to each bandit, is equal $\frac{1}{n+2} \binom{2n+2}{n+1}$, which is the $(n + 1)^{\text{th}}$ Catalan number, and the number of ways to complete the journey having at least one gold bar through the whole its duration is $\frac{1}{n+1} \binom{2n}{n}$ which equals the n^{th} Catalan number. In case of the modified problem, when the amount of bankers is equal the amount of bandits the number of ways to complete the journey is also the n^{th} Catalan number. The analysis shows that the methods used examine the problem from different perspectives, which allows deeper understanding of counting discrete structures.